SRI AKILANDESWARI WOMEN'S COLLEGE ,WANDIWASH

# OPERATION RESEARCH

## CLASS: III UG MATHEMATICS


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# WELCOME
# TO
# LINEAR PROGRAMMING PROBLEM AND ITS APPLICATION S

# INTRODUCTION

Linear programming (LP) is a tool for solving optimization problems.

In 1947, **George Dantzig** developed an efficient method, the simplex algorithm, for solving linear programming problems (also called LP).

The first formal presentation of the simplex technique was in the form of a paper published by Professor Dantzig in 1951.

In view of the competitive conditions prevailing in business and industry today, business-administration as well as engineering students must be exposed to the philosophy, techniques, and economic interpretation of linear programming methods.

The word 'linear' means the relationships handled are those represented by straight lines, i.e., the relationships are of the form $y = a + bx$ .The word 'programming' means taking decisions systematically.

- **CHARACTERISTICS:**
- The five characteristics of linear programming problem.

1. Constraints
2. Objective function
3. Linearity
4. Finiteness
5. Non-Negative

- More than one solution exist, the objectives being to select the optimum solution.

- **PRINCIPLES:**
- Proportionally
- Additivity
- Divisibility
- Certainty
- Optimality

- **USES:**

- Linear programming is used in the Food and agriculture industry where farmers can generate more revenues for their land for making profits.
- Manufacturing units work on this technique to generate more profit for the company.

- **ADVANTAGES:**

- Provide the best allocation of available resource.
- Put across overview points more successfully by logical argument supported by scientific method.

- **OPTIMAL SOLUTION**:

- An optimal solution is a feasible solution where the objective function reaches its maximum (or minimum) values.
- The values of x and y are said to be optimal solutions for which the objective function $z = x + y$ is minimum or maximum based on the given linear programming problems.

## ► INTEGER PROGRAMMING:

- A L.P.P in which solution requires is called an integer programming problems.

Example : Maximize

Subject to:

= (i= 1,2,……,m)

(j = 1,2,……,n)

integer (for some or all j = 1,2,….,n )

## ► GRAPHICAL LP SOLUTION

The graphical procedure includes two steps:

- Determine of the solution space the defines all feasible solutions of the model.
- Determine of the optimum solution from among all the feasible points in the solution space.

## ► FORMATION OF MATHEMATICAL MODEL OF L.P.P

There are three forms:

- General form of L.P.P
- Canonical form of L.P.P
- Standard form of L.P.P

# APPLICATION OF LINEAR PROGRAMMING PROBLEM

- Personal Assignment problem
- Transportation problem
- Proficiency in operation of Dam System
- Optimum Estimation of Executive Compensation
- Agricultural Application
- Military Application
- Production Management
- Marketing Management
- Manpower Management
- Physical distribution

# TYPES OF LINEAR PROGRAMMING PROBLEM

1. **GRAPHICAL METHOD:**

- A linear programing problem with only two variable presents a simple case, for which the solution can be derived using a graphical or geometrical method.
- Graphical method is one of the methods for solving linear programming problems. It includes the following steps:

- Formulate the problem and define it in simple mathematical equations.
- Plot the points and draw the lines accordingly.
- Identify the solution area.

**1.1 ADVANTAGES:**

- It makes data more easily understandable.
- Graphical methods are quick and easy to use and make visual sense.

## 1.2 LIMITATION

- The main drawback of the graphical approach of solving linear equations is that it cannot be used to solve problems with three or more variables.
- To answer issues involving graphical approximation, algebraic inequalities, applications of integrals, and areas of various forms using the graphical approach, one need to have a through understanding of the mathematics subject.

## 1.3 ALGORITHM

Consider the set of rectangular, the plane take the decision variable x on OX and y on OY. Then each point as coordinate of the type (x,y) and conversely.

**Step 1**: Any point (x,y) with stratify the condition $x \geq 0$, $y \geq 0$ lies in the first coordinate only, conversely for any point (x,y) in the first coordinate $(x,y) \geq 0$.

**Step 2**: Consider each inequality constraints an equation (straight line)

**Step 3:** Plot each straight line on the graph. The region in which each constraints holes when the inequality inactivated is indicated by the direction of the arrow mark on the associated straight line.

With the inequality constraints corresponding to the line is "≥" . The region above the line in the first coordinate is shaded. For the inequality ≤ the region below the line in the first coordinate is shaded. Then we have the common region in which all the point lying in it while satisfy all the constraints.

Our aim is to find a point in the feasible region optimize (maximize (or) minimize) the objective function.

**Step 4:** Assign an arbitrary value is a graph for the objective function.

**Step 5:** Draw the straight line to represent to objective function with the arbitrary value (the straight line through origin)

**Step 6:** In the minimization problem this line will be slope further away from the origin and passing through at least one corner of the feasible region. This is the point where the maxima is obtained. In the minimization problem nearest minima is obtained.

## 1.4 **EXAMPLE**:

Solve the following Graphically maximizing Z = X1+X2

Subject to

$X1 + 2X2 \le 2000$

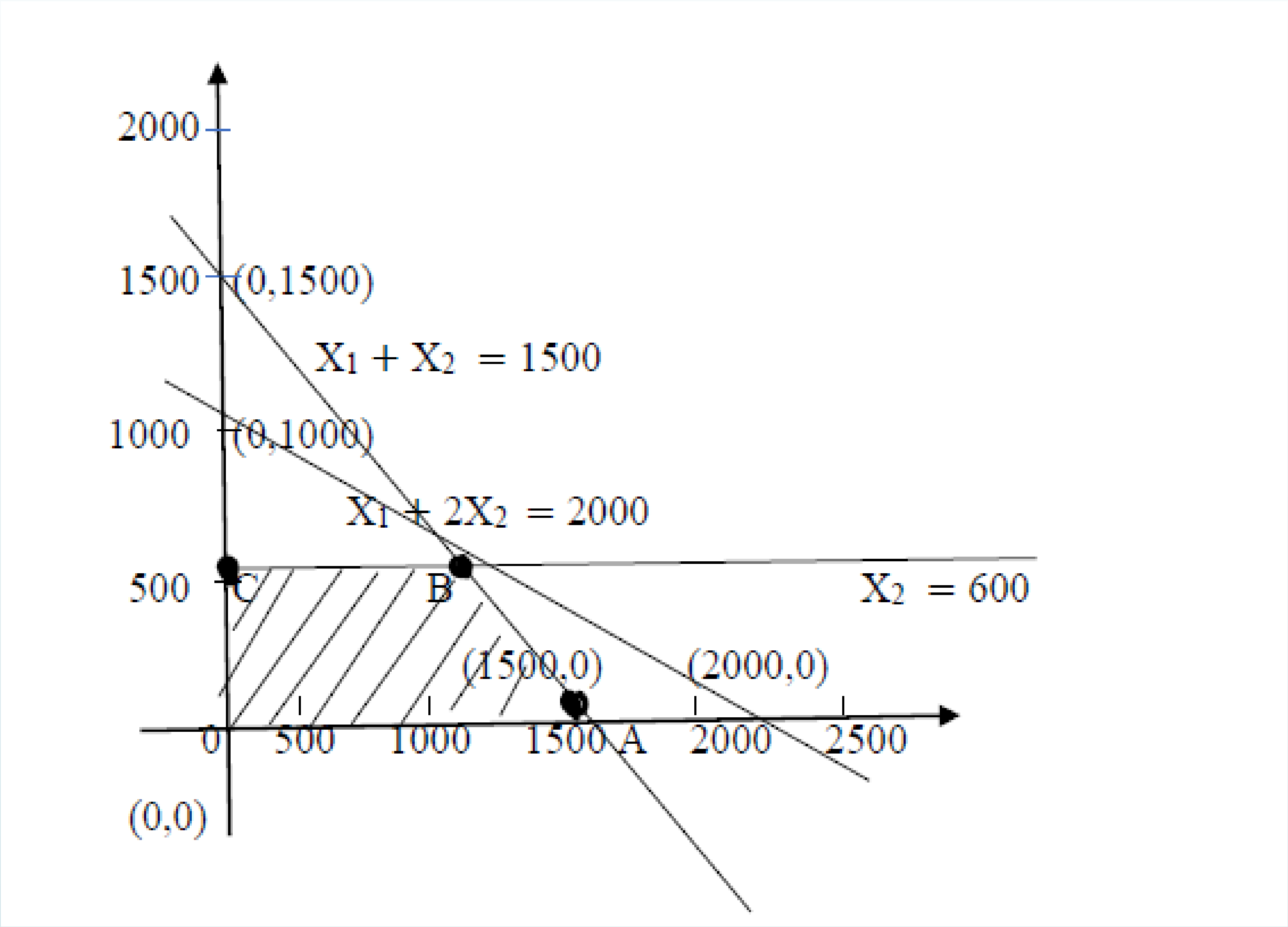
$X1 + X2 \le 1500$

$X2 \le 600$ and

$X1, X2 \ge 0.$

**Solution:**

| Corner Points | Maximize Point Z=$X_1$+$X_2$ |
|---|---|
| 0(0,0) | Z = 0+0 =0 |
| A(1500,0) | Z =1500+0 = 1500 (Maximize) |
| B(1000,500) | Z = 1000 +500 =1500 |
| C(800,600) | Z = 800+600 = 1400 |
| D(0,600) | Z = 0+600 = 600 |




Result:

The objective function as a maximum value 1500 at the point (1500,0)

Maximum Z= 1500 and the therefore the optimal solution is x = 1500, y = 0

## 2. SIMPLEX METHOD

- Simplex method is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solutions of an optimization problem.
- Simplex method is the steps of algorithm until an optimal solution is reached. It is also known as **iterative method.**

### 2.1 ADVANTAGE

- Efficiency
- Flexibility
- Optimization
- Decision Making
- Wide applicability

### 2.2 LIMITATION

- Nonlinear programming
- Problem Size
- Convergence issues
- Sensitivity to input data

## 2.3 ENUMERATION :

**Step 1:** Check whether the objective function of the given LPP is to be maximized (or) minimized. If it is to minimized then we convert it in to problems of maximizing by using the result.

Max() = -–mini (Z) or mini (−Z)

**Step 2:** Rewrite the given LPP is a standard form and get a starting basic feasible solution.

**Step 3:** OPTIMALITY TEST: Calculate $Z_j - C_j \geq 0$ (for maximize problem) the present solution is optimal if not all $Z_j - C_j \geq 0$ then the solution is not optimal, then go to next step.

**Step 4:** ENTERING VARIABLRS: The non- negative variable whose $Z_j - C_j$ is most negative is the entering variables. If the entering column does not contain any positive value then the given problem has an unbounded solution. Otherwise go to next step.

**Step 5:** LEAVING VARIABLE: Obtain the ratio of the solution of entering column

Only for a positive elements of entering column. The basic variable which have the minimum ratio which is the leaving variable.

**Step 6:** The intersection element of leaving row and entering column is called pivot element (key element).

The next simplex table is obtained by performing the following types.

**Type 1:**

New pivot row (or) key row =

**Type 2:**

New row = current row – ( Entering column coefficient) ( new pivot row)

**Step 7:** Goto step 3 and repeat the process until get a feasible solution.

## 2.4 EXAMPLE

**Solve the following LPP simplex method Maximize Z = $3x_1+9x_2$**

**Subject to**

**$X_1+4x_2$ 8**

**$X_1+2x_2$ 4 and**

**$X_1, X_2$ 0.**

## Solution:

**ITERATION TABLE :**

All the Co-efficient of the variable in the objective function row are non-negative.

The optimal solution Maz Z = 18

$X_2$= 0, $S_1$ = 0, $X_1$ = 0, $S_2$ = 0.

## 3. BIG M METHOD

In operation research, the Big-M method is a method of solving linear programming problems using the simplex algorithm. The Big-M method extends the simples algorithm to problems to contain " Greater-than" constraints. It does so by associating the constraints with large negative constants which would not be part of any optimal solution, if it exists. This is another method for solving linear programming problems, which has artificial variables.

### 3.1 ADVANTAGES

- The advantage to Big m-method is that the problem is solved in just one single phase.
- The two phase method, there's no guarantee of the quality of the initial solution to phase Two as the original objective is not considered at all in phase one.

### 3.2 LIMITATIONS :

- If M is too large, serious numerical difficulties in a computer
- Here feasibility is not known until optimality.

**3.3 ENUMERATION :**

The steps of this method are as follows:

**Step 1:**

Add slack variables, surplus variables and artificial variables to standard form as per the need of LP problem. Assign a very large value +M (in minimization) or −M (in maximization) to objective function.

**Step 2:**

Calculate all $C_j - Z_j$ of the last row.

a) If all calculated values are greater than or equal to zero, the solution is optimal .

b) Else, if any column has the most negative $C_j - Z_j$ values and all the corresponding entries in the column are negative, the solution is unbounded.

**Step 3:**

Follow the further steps of simplex method to solve.

**3.2 EXAMPLE :**

**Solve the following LPP using Big-M Method Max Z = $2X_1+5X_2$**

**Subject to**

**$X_1$ 40**

**$X_2$**

**$X_1,X_2$**

**$X_1,X_2$ 0**

**SOLUTION:**

Max Z = $2X_1+5X_2$

Subject to

$X_1+S_1 = 40$

$X_2+S_2 = 30$

$X_1+X_2-S_3 =60$ and

$X_1, X_2, S_1, S_2$

Introducing the artificial variable $A_1$ to the 3rd constraints.

Then the given problem can be written as

$$\text{Max } Z = 2X_1+5X_2-MA_1$$

Subject to

$$X_1+S_1 = 40$$

$$X_2+S_2 = 30$$

$$X_1+X_2-S_3+A_1 = 60$$

$$X_1, X_2, S_1, S_2, S_3, A_1$$

Initial basic feasible solution is

Put $X_1 = X_2 = S_3 = 0$ then $S_1 = 40$, $S_2 = 30$, $A_1 = 60$

To complete this problem we use three simplex table finally

## ITERATION TABLE:

| Basics | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | Solution |
|---|---|---|---|---|---|---|---|
| Z | 0 | 0 | 2 | 5 | 0 | M | 230 |
| $S_3$ | 0 | 0 | 1 | 1 | 1 | −1 | 10 |
| $X_2$ | 0 | 1 | 0 | 1 | 0 | 0 | 30 |
| $X_1$ | 1 | 0 | 1 | 0 | 0 | 0 | 40 |

All the co-efficient of variables in the objective row are non-negative.

$$A_1 = A_2 = S_1 = S_2 = 0$$

$$\text{Max } Z = 280$$

$$\text{Max } Z = 2X_1 + 5X_2$$

$$2(40)+5(30)$$

$$= 80 + 150$$

$$\text{Max } Z = 230.$$

## 4. TWO PHASE METHOD

- Two Phase Method is another method of solving a LPP that involve artificial variables.
- The solution is obtained in two phases. In phase1, the sum of the artificial variables is minimized subject to the given constraints of the given LPP. If this solution contains artificial variables with positive value then the given LPP has no optimum solution.
- Otherwise, we get to Phase2, where the basic feasible solution obtained in Phase 1 is taken as the initial basic feasible solution to the given LPP and then the simplex algorithm is aoolied.

### 4.1 BENEFITS:

- Two phase method eliminates the artificial variables in the beginning in the phase one itself the artificial variables are eliminated.
- It is also able to handle problems with inequalities and non-negative variables, which cannot be solved using other methods such as the simplex method.

### 4.2 LIMITATION:

- One limitation of the two phase methods is that it may require more iterations to find the optimal solution compared to other methods.
- It also relies on the existence of an initial feasible solution, which may be difficult to find for some problems.

## 4.1 ENUMERATION :

**PHASE I.**

**Step 1 :**

Let $Z'$ be the new objective function obtained by replacing all the co-efficient $c_1$ of variables $x_i$ to zero in $z = cx$ and introducing required artificial variables $a_i$ with co-efficient -1 for each. The LPP given by maximize $Z'$ subject to the constraints of the given LPP is known as **auxiliary LPP.**

**STEP 2 :**

Solve the auxiliary LPP by applying the simplex algorithm. Then the following cases may arise.

**CASE 1:** Maximum $z' = 0$ and at least one artificial variable is present in the basis with positive values. Then the original LPP has no optimal solution.

**CASE 2:** Maximum $z' = 0$ and no artificial variable is present or an artificial variable is present with zero value then we go to phase II.

**PHASE II.**

Consider the original objective function $z = cx$. If, in the basic feasible solution obtained in Phase I, the artificial variable with zero value is present then the artificial variable is added to $z'$ with zero co-efficient . then solve the LPP using simplex algorithm with the basic feasible solution obtained in Phase I as the starting solution.

**EXAMPLE :1**

**Solve the LPP Two phase method**

**Minimize** $Z = 2X_1+4X_2$

**Subject to**

$2X_1+X_2 \geq 4$

$X_1+2X_2$ 3

$X_1,X_2$ 0.

**SOLUTION**

**PHASE 1:**

## PHASE 2:

Since all $Z_j$ $C_j$ it is an optimal solution.

Values of Variable are

$X_1$ =

$X_2$ = and

Max Z = −6

Min Z = 6

## 5. DUAL METHOD

Each Linear programming problem (either maximization or minimization ) stated in its original form is associated with another unique linear programming problem, based on the same data. In general, it is immaterial which of the two problems is called primal or dual.

### 5.1 GENERAL CANON FOR CONVERTING ANY PRIMAL INTO IT'S DUAL:

If the system of constraints in a given LPP of a mixture of equalities, inequalities non-negative variables or and unrestricted variables.

Then the dual of the given problem uptrained by reducing to standard primal form by the following **STEPS :**

**STEP 1 :**

First convert to the objective function to maximization from if not.

**STEP 2:**

If a constraints as inequalities sign then multiple on both side and made the inequalities sign .

**STEP3:**

If the constraints as an quality sign then it is replaced by two constraints involving the inequalities going in opposite direction.

**STEP 4:**

Every unrestricted variable is replaced by the difference of two non- negative variable.

**STEP 5:**

We get the standard form of given LPP in which

(a) All the constraints have sign where the objective form is maximization form.

(b) All the constraints have sign where the objective form is minimization form.

**STEP 6:**

Finally the dual of the given problem is obtained by

(a) Transporting rows & columns of constraints co-efficient.

(b) Transporting the co-efficient ($c_1, c_2, \ldots\ldots, c_n$) of the objective function and the right side constant ($b_1, b_2, \ldots\ldots, b_n$).

(c ) changing the inequalities form sign.

(d) Minimizing the objective function instead of maximization it.

# LIMITATION OF LINEAR PROGRAMMING

- There is a possibility that both function are linear. Determining the given function automatically in a linear programming problem is quite difficult.
- The solutions obtained can be real numbers all the time.
- All the constrains and coefficients are mentioned in linear programming with certainty. We can compute the solution manually if number of variables or constraints are very large.
- When the objective function is determined, it is not easy to find social, institutional, and other constraints
- The limitations of linear programming are if we assume that all relation are linear, then it may not hold good for all the situations.

# THANK YOU